Supplementary Document: Real-time Rendering of Layered Materials with Anisotropic Normal Distributions

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A ADDITIONAL FORMULAS

A.1 Derivation of Jacobian Matrices

To derive Jacobian matrices, we partly followed the derivation by Stam [2001]. Different from his derivation, we derived an approximate solution for the Jacobian matrices over the region near to the origin of $\mathcal{P}$, while Stam derived the exact solution only at the origin. Without loss of generality, we can assume incident direction $\boldsymbol{\omega}_i$ as $(\theta_i, 0)$. Let $\boldsymbol{\omega}_r$ and $\boldsymbol{\omega}_t$ be directions for reflection and refraction, respectively. We denote the directions $\boldsymbol{\omega}_i$, $\boldsymbol{\omega}_r$, $\boldsymbol{\omega}_t$, and $\mathbf{h}$ as follows:

$$\begin{aligned}
\boldsymbol{\omega}_i &= (\sin\theta_i, 0, \cos\theta_i), \\
\boldsymbol{\omega}_r &= (x_r, y_r, z_r), \\
\boldsymbol{\omega}_t &= (x_t, y_t, z_t), \\
\mathbf{h} &= (x_h, y_h, z_h).
\end{aligned}$$

Let η be a relative refractive index between two interfaces, we can write $\boldsymbol{\omega}_r$ and $\boldsymbol{\omega}_t$ as follows:

$$\begin{aligned}
\boldsymbol{\omega}_r &= 2(\boldsymbol{\omega}_i \cdot \mathbf{h})\mathbf{h} - \boldsymbol{\omega}_i, \\
\eta\boldsymbol{\omega}_t &= \left(\boldsymbol{\omega}_i \cdot \mathbf{h} - \sqrt{(\boldsymbol{\omega}_i \cdot \mathbf{h})^2 + \eta^2 - 1}\right)\mathbf{h} - \boldsymbol{\omega}_i.
\end{aligned}$$

Using these equations, we can obtain the projected 2D coordinates (x_r, y_r) and (x_t, y_t) of $\boldsymbol{\omega}_r$ and $\boldsymbol{\omega}_t$:

$$\begin{cases}
x_r = 2Ax_h - \sin\theta_i \\
y_r = 2Ay_h
\end{cases}$$
$$\begin{cases}
\eta x_t = \left(A - \sqrt{A^2 + \eta^2 - 1}\right) x_h - \sin\theta_i \\
\eta y_t = \left(A - \sqrt{A^2 + \eta^2 - 1}\right) y_h
\end{cases}$$
$$\text{where} \quad A = x_h \sin\theta_i + \cos\theta_i \sqrt{1 - x_h^2 - y_h^2}.$$

Therefore, for reflection, the Jacobian matrix is obtained as in the main body of the paper. For refraction, the Jacobian matrix is calculated as follows:

$$\mathbf{J}_t = \begin{bmatrix} \frac{\partial x_t}{\partial x_h} & \frac{\partial x_t}{\partial y_h} \\ \frac{\partial y_t}{\partial x_h} & \frac{\partial y_t}{\partial y_h} \end{bmatrix},$$

$$\begin{aligned}
\eta\frac{\partial x_t}{\partial x_h} &= A - \sqrt{A^2 + \eta^2 - 1} + x_h \frac{\partial A}{\partial x_h}\left(1 - \frac{2A}{\sqrt{A^2 + \eta^2 - 1}}\right) \\
\eta\frac{\partial x_t}{\partial y_h} &= x_h \frac{\partial A}{\partial y_h}\left(1 - \frac{2A}{\sqrt{A^2 + \eta^2 - 1}}\right) \\
\eta\frac{\partial y_t}{\partial x_h} &= y_h \frac{\partial A}{\partial x_h}\left(1 - \frac{2A}{\sqrt{A^2 + \eta^2 - 1}}\right) \\
\eta\frac{\partial y_t}{\partial y_h} &= A - \sqrt{A^2 + \eta^2 - 1} + y_h \frac{\partial A}{\partial y_h}\left(1 - \frac{2A}{\sqrt{A^2 + \eta^2 - 1}}\right)
\end{aligned}$$
$$\text{where} \begin{cases}
\frac{\partial A}{\partial x_h} = \sin\theta_i - \frac{x_h \cos\theta_i}{\sqrt{1 - x_h^2 - y_h^2}}, \\
\frac{\partial A}{\partial y_h} = -\frac{y_h \cos\theta_i}{\sqrt{1 - x_h^2 - y_h^2}}
\end{cases}$$

As we wrote in the main body of the paper, we assume x_h, y_h, and θ_i are small enough that we can ignore the second- and higher-order terms of x_h, y_h, and $\sin\theta_i$. Then, we can approximate $\mathbf{J}_t$ as follows:

$$\begin{aligned}
\mathbf{J}_t &\approx \frac{1}{\eta}\begin{bmatrix} \cos\theta_i - \sqrt{\cos^2\theta_i + \eta^2 - 1} & 0 \\ 0 & \cos\theta_i - \sqrt{\cos^2\theta_i + \eta^2 - 1} \end{bmatrix} \\
&= \frac{1}{\eta}\begin{bmatrix} \cos\theta_i - \cos\theta_t & 0 \\ 0 & \cos\theta_i - \cos\theta_t \end{bmatrix}
\end{aligned}$$

Thus, the Jacobian matrix for refraction is also diagonal and its diagonal entries are the same.

A.2 Adding-Doubling for Two-layer Materials

For two-layer materials, Belcour [2018] provided the result of the adding-doubling method in Section 5 of his paper. To extend their formulas using our result for anisotropic distribution is easy. By replacing the scalar variances $\sigma_{ij}^{\{T,R\}}$ with covariance matrices $\Sigma_{ij}^{\{T,R\}}$. The series of interactions that are possible in two-layer materials are only R and $TR^{+}T$. The atomic operators for R are given by

$$\begin{aligned}
e^R &= r_{12}, \\
\boldsymbol{\mu}^R &= -\boldsymbol{\mu}_i, \\
&\boxed{\Sigma^R = r_{12}\Sigma_{12}^R}.
\end{aligned}$$

For TR^+T, the atomic operators are obtained as follows:

$$e^{TR^+T} = \frac{t_{12}r_{23}t_{12}}{1 - r_{23}r_{12}},$$

$$\boldsymbol{\mu}^{TR^+T} = -\boldsymbol{\mu}_i,$$

$$\boxed{\Sigma^{TR^+T} = \frac{t_{12}r_{23}t_{12}}{1 - r_{23}r_{12}}\left[\Sigma_{12}^{T} + \Sigma_{21}^{T} + K_{21}\left(\Sigma_{23}^{R} + \frac{r_{23}r_{21}}{1 - r_{23}r_{21}}\Sigma_{21}^{R}\right)\right]}.$$

In these formulas, r_{jk} and t_{jk} denote reflection and transmission coefficients between j-th and k-th interfaces, and K_{jk} is a transmission scaling factor which scales the roughness parameters. As explained in the main body of the paper, $\Sigma_{12}^{\{R,T\}}$ can be obtained as follows:

$$\Sigma_{12}^{\{R,T\}} = \begin{bmatrix}\mathbf{t}_x & \mathbf{t}_y\end{bmatrix}^{\top} \begin{bmatrix}\sigma_{12,x}^{\{R,T\}} & 0 \\ 0 & \sigma_{12,y}^{\{R,T\}}\end{bmatrix} \begin{bmatrix}\mathbf{t}_x & \mathbf{t}_y\end{bmatrix},$$

$$\sigma_{12,\{x,y\}}^{R} = h\left(\alpha_{\{x,y\}}\right), \quad \sigma_{12,\{x,y\}}^{T} = h\left(s \times \alpha_{\{x,y\}}\right).$$

REFERENCES

L. Belcour. 2018. Efficient rendering of layered materials using an atomic decomposition with statistical operators. *ACM Trans. Graph.* 37, 4, Article 73 (2018), 15 pages. https://doi.org/10.1145/3197517.3201289

J. Stam. 2001. An illumination model for a skin layer bounded by rough surfaces. In *Eurographics Workshop on Rendering*. 39–52. https://doi.org/10.2312/EGWR/EGWR01/039-052

(Appendix B starts from the next page)

B ADDITIONAL RESULTS

B.1 Results for varying roughness parameters

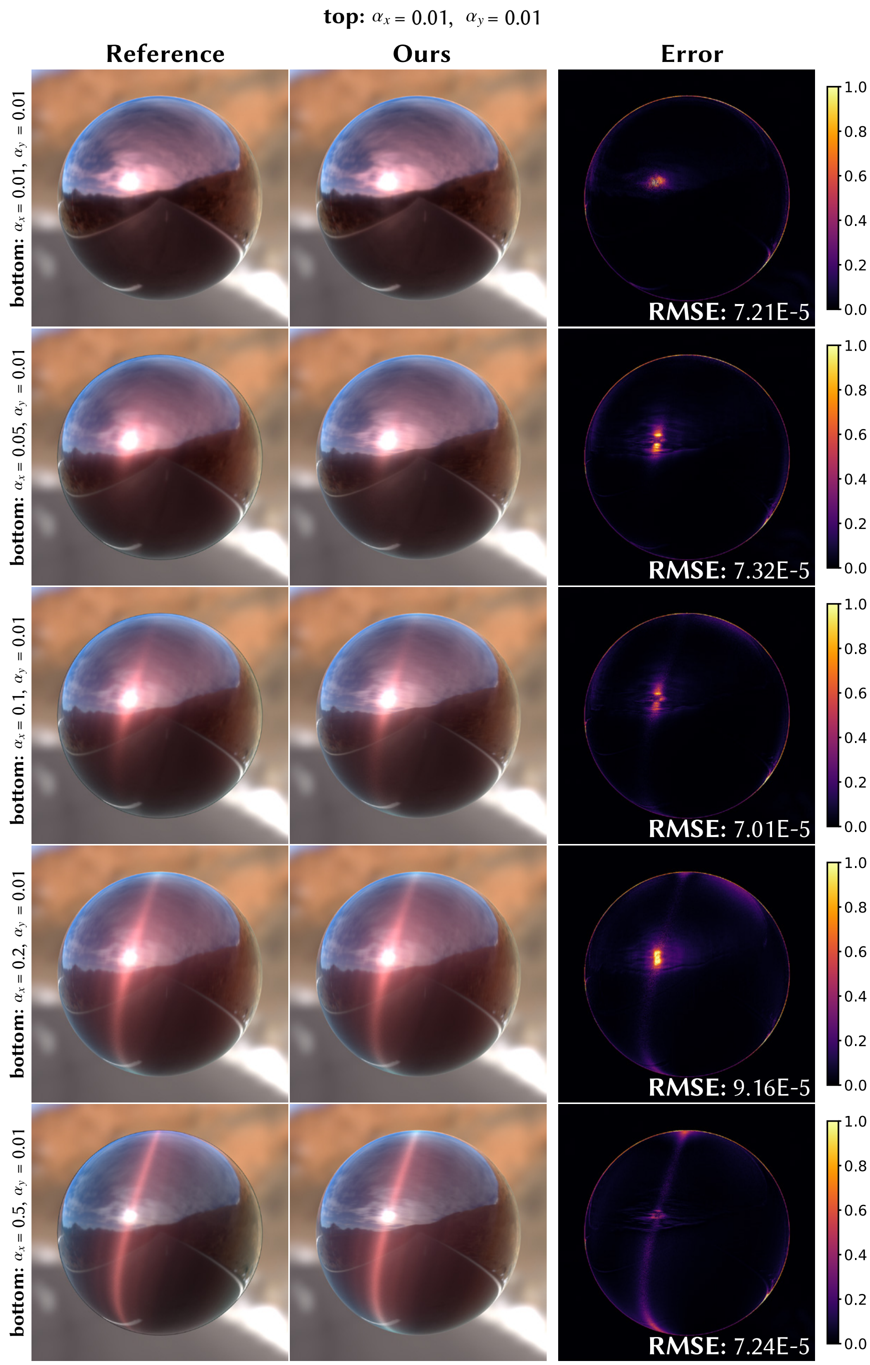

Figure 1: Rendering results with varying roughness parameters on the bottom layer ranging from 0.01 **to** 0.5**. The roughness parameters of the top layer are fixed at**$(\alpha_x, \alpha_y) = (0.01, 0.01)$

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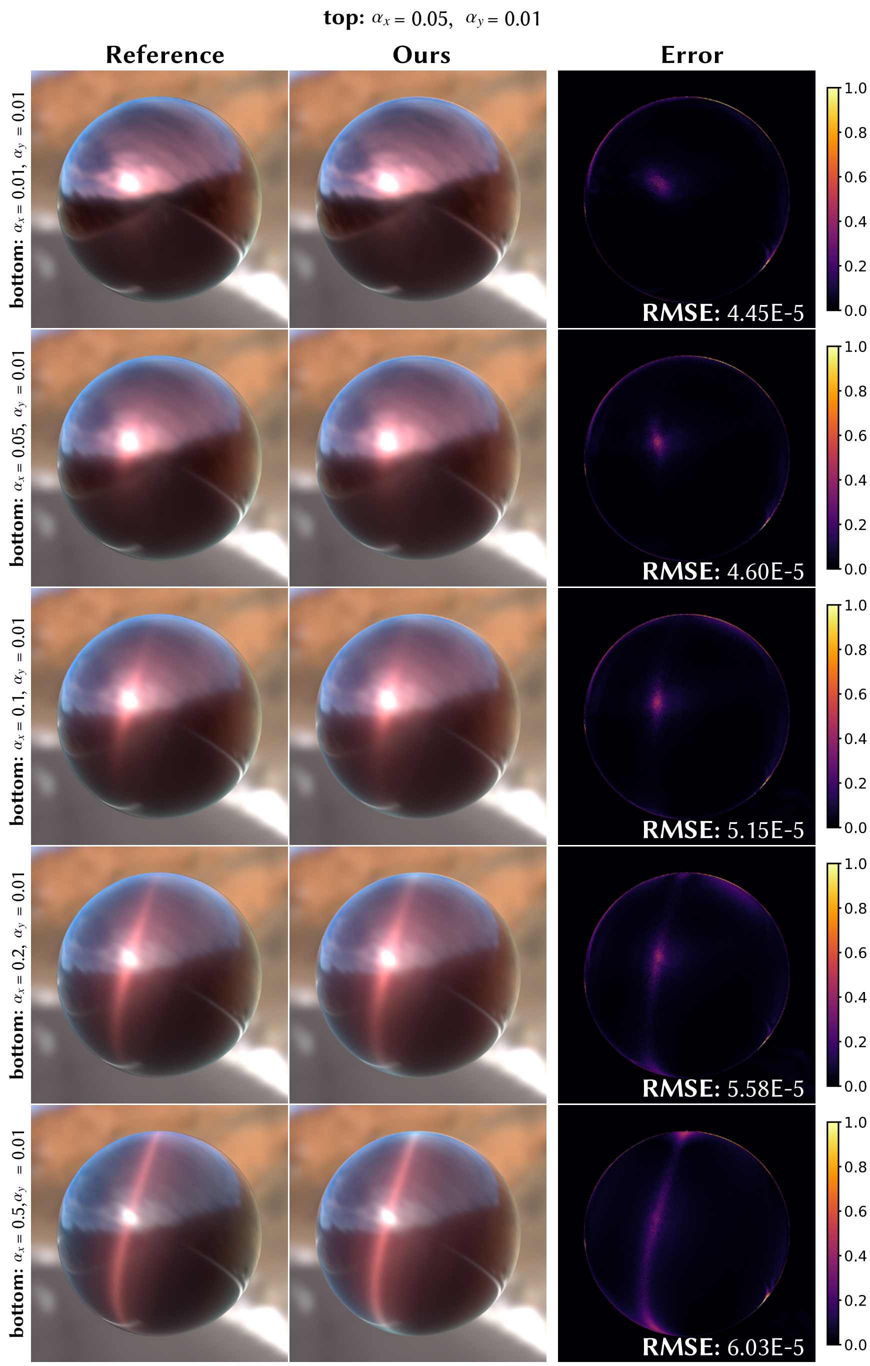

Figure 2: Rendering results with varying roughness parameters on the bottom layer ranging from 0.01 to 0.5. The roughness parameters of the top layer are fixed at $(\alpha_x, \alpha_y) = (0.05, 0.01)$

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B.2 Results for varying rotation of local coordinate system

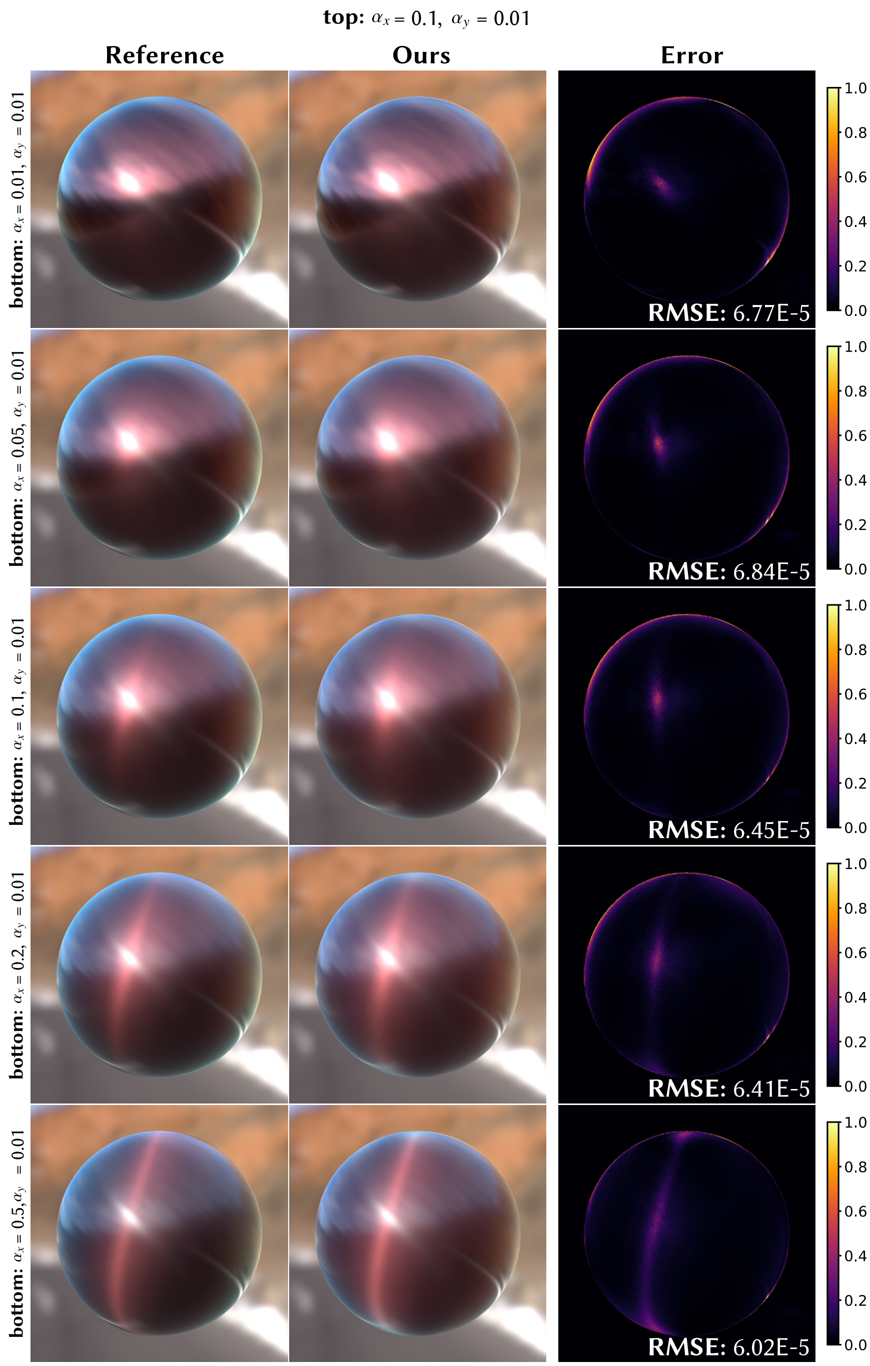

Figure 3: Rendering results with varying roughness parameters on the bottom layer ranging from 0.01 to 0.5. The roughness parameters of the top layer are fixed at $(\alpha_x, \alpha_y) = (0.1, 0.01)$

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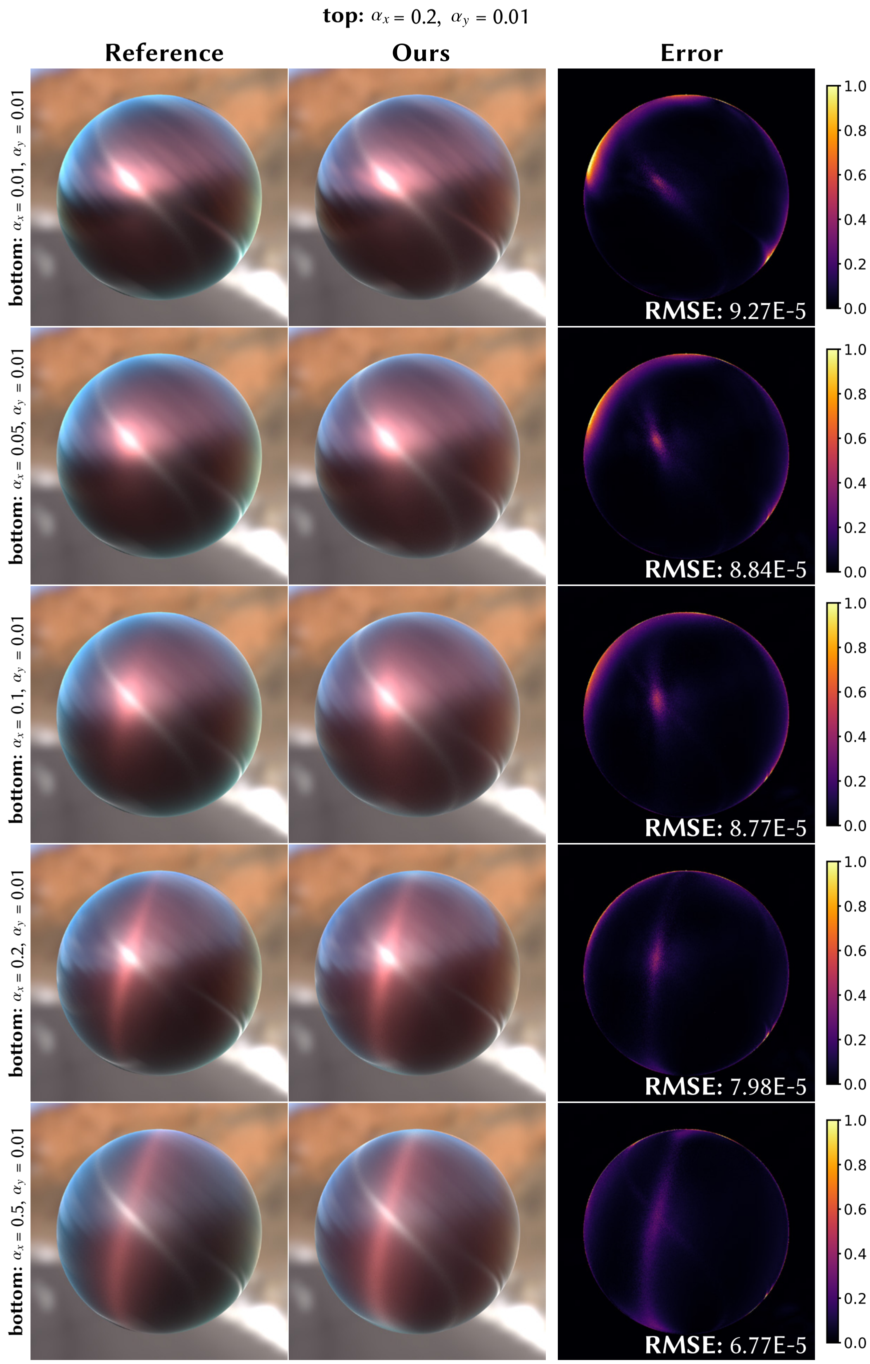

Figure 4: Rendering results with varying roughness parameters on the bottom layer ranging from 0.01 to 0.5. The roughness parameters of the top layer are fixed at $(\alpha_x, \alpha_y) = (0.2, 0.01)$

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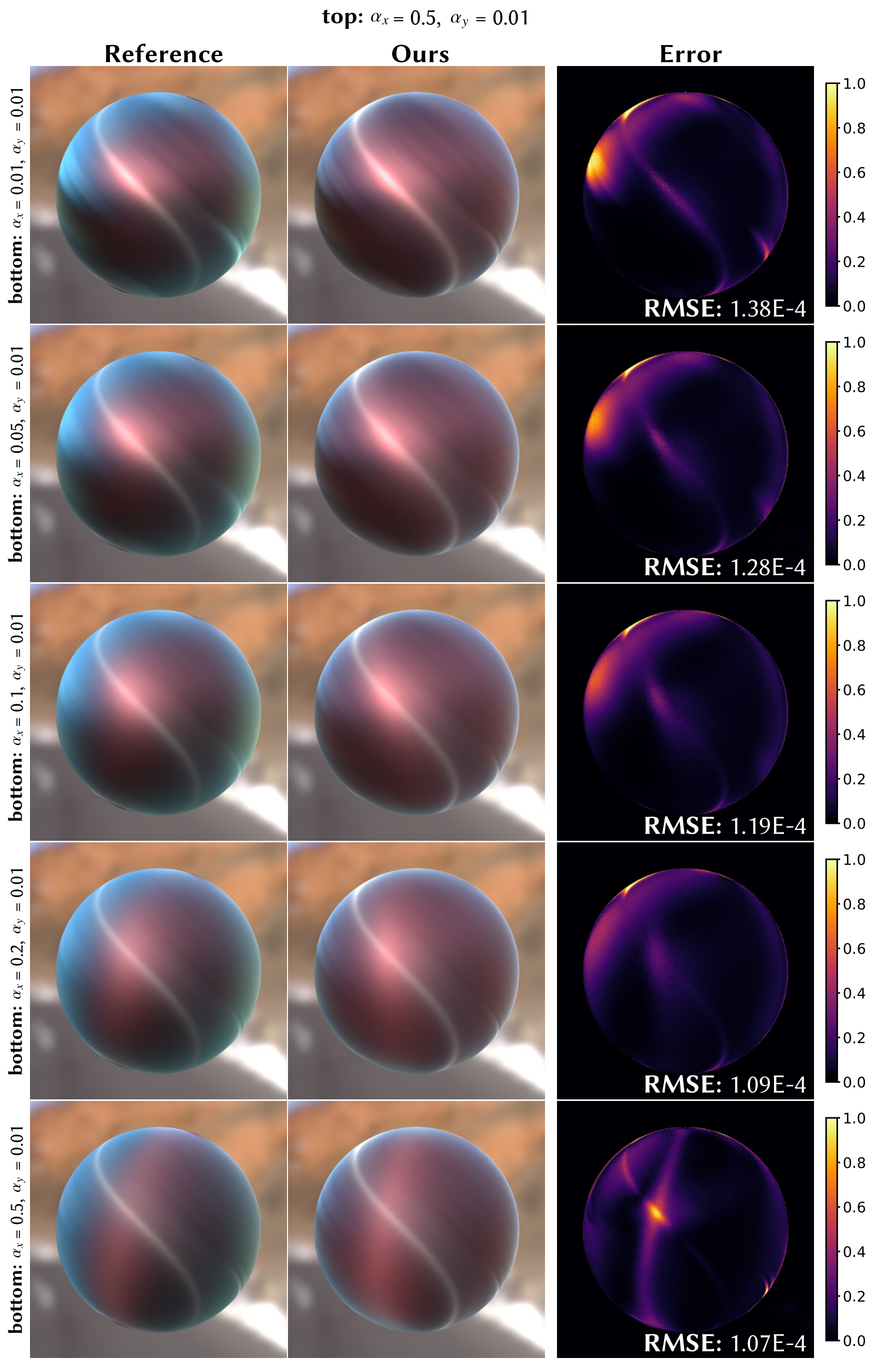

Figure 5: Rendering results with varying roughness parameters on the bottom layer ranging from 0.01 to 0.5. The roughness parameters of the top layer are fixed at $(\alpha_x, \alpha_y) = (0.5, 0.01)$

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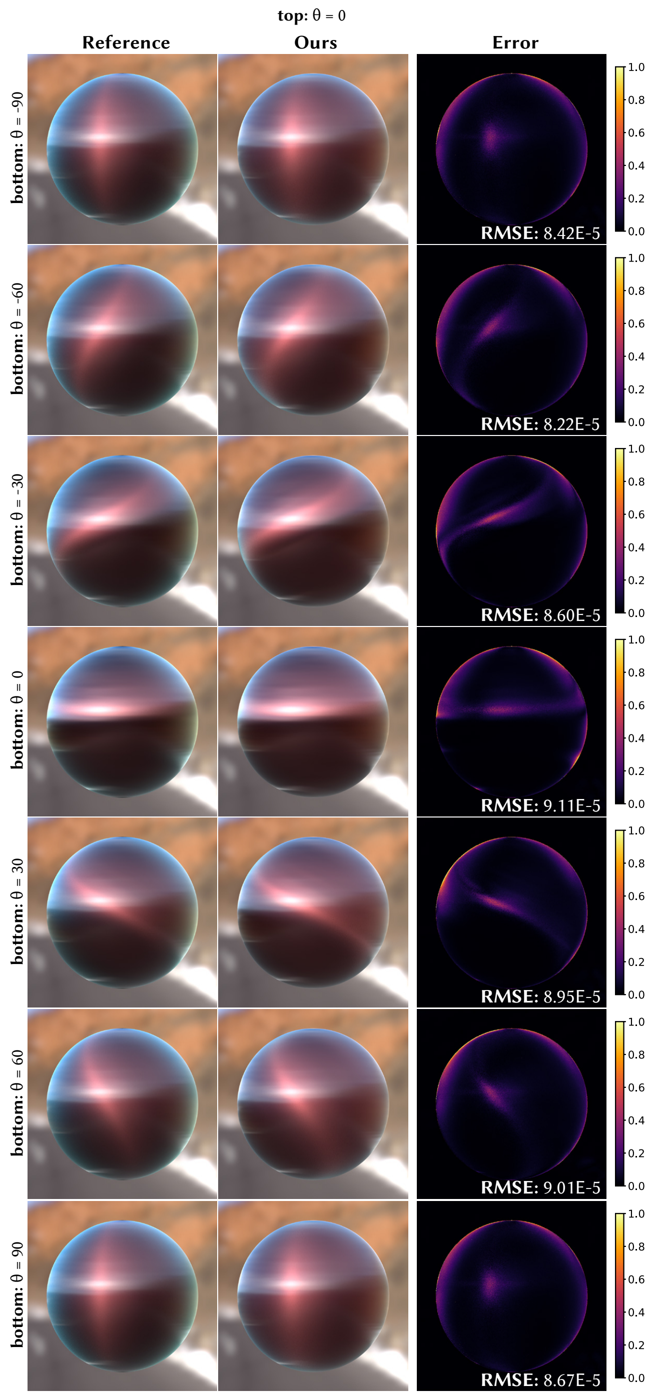

Figure 6: Rendering results for rotated local coordinate systems for the *bottom* layer, while the local coordinate system of the top layer is fixed.

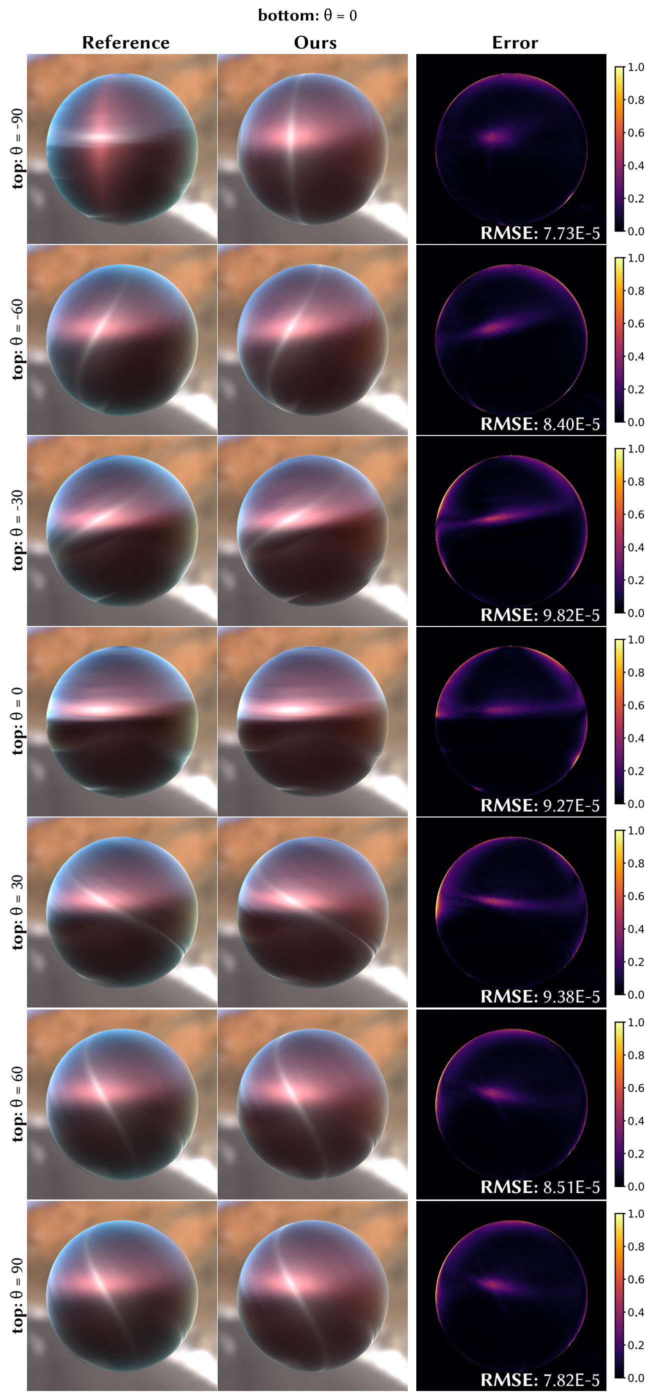

Figure 7: Rendering results for rotated local coordinate systems for the *top* layer, while the local coordinate system of the bottom layer is fixed.